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2008 J. Phys. A: Math. Theor. 41 164051

(http://iopscience.iop.org/1751-8121/41/16/164051)

IOP PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND THEORETICAL

J. Phys. A: Math. Theor. **41** (2008) 164051 (8pp) doi:10.1088/1751-8113/41/16/164051

The spontaneous creation of a chromomagnetic field and A_0-condensate at high temperature on a lattice

V Demchik and V Skalozub

Dnipropetrovsk National University, Dnipropetrovsk, Ukraine

E-mail: vadimdi@yahoo.com and skalozub@ff.dsu.dp.ua

Received 25 October 2007, in final form 13 January 2008
Published 9 April 2008
Online at stacks.iop.org/JPhysA/41/164051

Abstract

In a lattice formulation of $SU(2)$-gluodynamics, the spontaneous generation of chromomagnetic fields at high temperature is investigated. A procedure to determine this phenomenon is developed. By means of the χ^2-analysis of the data set accumulating 5–10 × 10^6 Monte Carlo configurations, the spontaneous creation of the Abelian color magnetic field is indicated. The common generation of the magnetic field and A_0-condensate is also studied. It is discovered that the field configuration consisting of the magnetized vacuum and the A_0-condensate is stable.

PACS numbers: 12.38.Gc, 98.62.En, 98.80.Cq

1. Introduction

Nowadays it is generally accepted that the nonlinearity of non-Abelian gauge fields could result in the formation of field condensates. The first model of a gluon condensate, the so-called '*color ferromagnetic vacuum state*', was proposed 30 years ago by Savvidy [1]. It describes the spontaneous generation of the uniform Abelian chromomagnetic field $H = \text{const}$ due to a vacuum polarization. Unfortunately, this state is unstable because of the tachyonic mode in the gluon spectrum [2, 3]. This situation is changed at finite temperature when the spectrum stabilization happens due to either a gluon magnetic mass [6] or a so-called A_0-condensate [5] that is implemented in a stable magnetized vacuum. These are the extensions of the Savvidy model to the finite temperature case already investigated by the methods of continuum field theory [4–6]. In these ways the possibility of the spontaneous generation of strong temperature-dependent and stable color magnetic fields of order $gH \sim g^4T^2$, where g is a gauge coupling constant and T is the temperature, is realized. The field stabilization is ensured by the temperature- and field-dependent gluon magnetic mass, which serves as a regulator of infrared singularities at $T \neq 0$.

Such spontaneously created 'primordial' color magnetic fields had been generated at a GUT scale [7]. They could serve as seed fields responsible for the generation of the large-scale magnetic fields detected in astrophysical observations [8]. The presence of strong magnetic fields in the early universe is of paramount importance for its evolution. In particular, a present day cosmic magnetic field of order $B_0 \sim (4\text{–}5) \times 10^{-9} G$ could produce the recently discovered Wilkinson microscopic anisotropy probe (WMAP) anomaly [9].

In [6, 10] the spontaneous creation of the chromomagnetic fields was observed in $SU(2)$- and $SU(3)$-gluodynamics within the one-loop plus daisy resummations. In [11] the chromomagnetic condensate of the same order was calculated in a stochastic QCD vacuum model by comparison with some data of lattice simulations. In [12] the effect on spontaneous generation of the chromomagnetic field at high temperature was checked in a lattice formulation of $SU(2)$-gluodynamics (which straightly has relevance to a weak isospin). In [13] the response of the vacuum to the influence of strong external fields at different temperatures was investigated, and it has been shown that confinement is restored when the strength of the external field is increased.

The other condensate, whose formation at $T \neq 0$ was investigated in continuum field theory [14], is the zero component of the gluon electrostatic potential $A_0 = \text{const}$. It acts to stabilize the QCD vacuum magnetized state, as was discussed in [5]. In this paper, however, the magnetic field was considered as an external one. Whether or not the actual value of the A_0 generated in the deconfining phase is sufficient to remove the tachyonic instability was not estimated.

The main goal of the present paper is to investigate the common generation of the chromomagnetic field and A_0-condensate in lattice simulations. To incorporate the chromomagnetic field on a lattice, we use the method developed in [12]. Instead of the field strength, which is quantized, the magnetic fluxes are considered as the objects to be investigated. The flux takes continuous values. Therefore, the minimization of free energy of the flux can be done in a usual known procedure. The values of the strength of a spontaneously generated magnetic field and the values of the Polyakov loop for corresponding states were obtained from Monte Carlo (MC) simulations. Then the value of A_0 was derived by using a procedure developed in [15].

The paper is organized as follows. In section 2, necessary information about the chromomagnetic fluxes on a lattice is educed and the results of calculations are given. In section 3, the investigation of the effect of a combined generation of the chromomagnetic field and A_0-condensate is presented. The final section is devoted to a discussion.

2. Chromomagnetic fields on a lattice

In continuum, to determine the spontaneous generation of a magnetic field one has to minimize the effective potential (or free energy) of the background field. The background field is introduced by splitting of the gauge field potential A_μ into the quantum A_μ^R and classical $\bar{A_\mu}$ parts: $A_\mu = A_\mu^R + \bar{A_\mu}$. We choose the potential $\bar{A_\mu}^a = (0, 0, Hx^1, 0)\delta^{a3}$ that corresponds to a constant chromomagnetic field directed along the third axis in the Euclidean and color space.

We relate the free energy density of the flux to the lattice action according to the definition

$$F(\varphi) = \bar{S}(\varphi) - \bar{S}(0), \qquad (1)$$

where $\bar{S}(\varphi)$ and $\bar{S}(0)$ are the lattice actions with and without the chromomagnetic field, respectively, and φ is the field flux.

To detect the spontaneous creation of the field, it is necessary to show that free energy has a global minimum at non-zero flux, $\varphi_{\min} \neq 0$.

In what follows, we use the hypercubic lattice $L_t \times L_s^3$ $(L_t < L_s)$ with the hypertorus geometry; L_t and L_s are the temporal and the spatial sizes of the lattice, respectively. In the

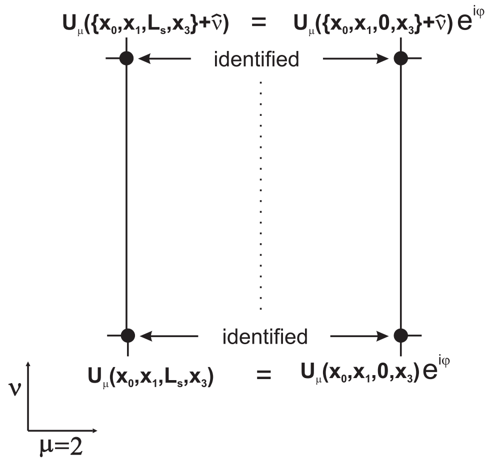

Figure 1. The plaquette presentation of the twisted boundary conditions.

limit of $L_s \to \infty$, the temporal size L_t is related to physical temperature. The standard Wilson action of the $SU(2)$ lattice gauge theory can be written as

$$S_W = \beta \sum_x \sum_{\mu>\nu} \left[1 - \frac{1}{2}\operatorname{Tr} U_{\mu\nu}(x)\right] \tag{2}$$

$$U_{\mu\nu}(x) = U_\mu(x)U_\nu(x + a\hat{\mu})U_\mu^\dagger(x + a\hat{\nu})U_\nu^\dagger(x), \tag{3}$$

where $\beta = 4/g^2$ is the lattice coupling constant, g is the bare coupling, $U_\mu(x)$ is the link variable located on the link leaving the lattice site x in the μ direction and $U_{\mu\nu}(x)$ is the ordered product of the link variables. The action $\bar{S}$ in (1) is the Wilson action S_W averaged over the Boltzmann configurations produced in the MC simulations.

The lattice variable $U_\mu(x)$ can be decomposed in terms of the unity, I, and Pauli, σ_j, matrices in the color space:

$$U_\mu(x) = IU_\mu^0(x) + \mathrm{i}\sigma_j U_\mu^j(x). \tag{4}$$

The chromomagnetic flux φ through the whole lattice was introduced by applying the twisted boundary conditions [16] (figure 1). In this approach the edge links in all directions are identified as usual periodic boundary conditions except for the links in the second spatial direction, for which the additional phase φ is added. The magnetic flux φ is measured in angular units and can take a value from 0 to 2π.

The twisted boundary conditions (t.b.c.) for the components of (4) are

$$U_\mu^0(x) \leftrightarrow \begin{cases} U_\mu^0 \cos\varphi - U_\mu^3 \sin\varphi & \text{for} \quad x = \{x_0, x_1, L_s, x_3\} \text{ and } \mu = 2, \\ U_\mu^0 & \text{for other links,} \end{cases} \tag{5}$$

$$U_\mu^3(x) \leftrightarrow \begin{cases} U_\mu^0 \sin\varphi + U_\mu^3 \cos\varphi & \text{for} \quad x = \{x_0, x_1, L_s, x_3\} \text{ and } \mu = 2, \\ U_\mu^3 & \text{for other links.} \end{cases} \tag{6}$$

Relations (5) and (6) have been implemented into the kernel of the MC procedure in order to produce the configurations with the chromomagnetic flux φ. Thus, the flux φ is taken into account while obtaining a Boltzmann ensemble at each MC iteration.

Table 1. The values of the generated fluxes φ_{min} for different lattices.

	2×8^3	2×16^3	4×8^3
$\beta = 3.0$	$0.019^{+0.013}_{-0.012}$	$0.0069^{+0.0022}_{-0.0057}$	$0.005^{+0.005}_{-0.003}$
$\beta = 5.0$	$0.020^{+0.011}_{-0.010}$		

The MC simulations are carried out by means of the heat bath method. The lattices $2 \times 8^3, 2 \times 16^3$ and 4×8^3 at $\beta = 3.0, 5.0$ are considered. These values of the coupling constant correspond to the deconfinement phase and perturbative regime. To thermalize the system, 200–500 iterations are fulfilled. At each working iteration, the plaquette value (3) is averaged over the whole lattice leading to the Wilson action (2). Then the action is calculated by averaging over the 1000–5000 working iterations. By setting a set of chromomagnetic fluxes φ in the MC simulations, we obtain the corresponding set of values of the action. The value of the condensed chromomagnetic flux φ_{min} is obtained as the result of minimization of the free energy density (1) in φ.

The spontaneous generation of the chromomagnetic field is the effect of order $\sim g^4$ [6]. The results of MC simulations show the comparably large dispersion. So, the large amount of the MC data is collected and the standard χ^2-method for the analysis of data is applied to determine the effect. We consider the results of the MC simulations as observed 'experimental data'.

The action depends smoothly on the flux φ in the region $\varphi \sim 0$. So, the free energy density can be fitted by a quadratic function of φ:

$$F(\varphi) = F_{\text{min}} + b(\varphi - \varphi_{\text{min}})^2. \tag{7}$$

In equation (7), there are three unknown parameters, F_{min}, b and φ_{min}. φ_{min} denotes the minimum position of free energy, whereas F_{min} and b are the free energy density at the minimum and the curvature of the free energy function, correspondingly.

The value φ_{min} is obtained as a result of minimization of the χ^2-function:

$$\chi^2(F_{\text{min}}, b, \varphi_{\text{min}}) = \sum_i \frac{(F_{\text{min}} + b(\varphi_i - \varphi_{\text{min}})^2 - F(\varphi_i))^2}{D(F(\varphi_i))}, \tag{8}$$

$$D(F(\varphi_i)) = \sum_{i \in \text{bin}} \frac{(F(\varphi_i) - \hat{F}_{\text{bin}})^2}{n_{\text{bin}} - 1}, \tag{9}$$

where φ_i is the array of the set of fluxes and $D(F(\varphi_i))$ is the data dispersion, which can be obtained by collecting the data into bins (as a function of φ), n_{bin} is the number of points and and $\hat{F}_{\text{bin}}$ is a mean value of free energy density in the considered bin. As was determined in the data analysis, the dispersion is independent of φ. The deviation of φ_{min} from zero indicates the presence of the spontaneously generated field.

The fit results are given in table 1. As one can see, φ_{min} demonstrates the 2σ-deviation from zero.

The 95% confidence level domain of parameters F_{min} (b for the right figure) and φ_{min} is represented in figure 2. The black cross marks the position of the maximum-likelihood values of F_{min} (b for the right figure) and φ_{min}. It is seen that the flux φ_{min} and other parameters F_{min} and b are positively determined. The 95% C.L. area becomes more symmetric with the center at F_{min}, b and φ_{min} when the statistics is increased. This also confirms the results of the fitting carried out.

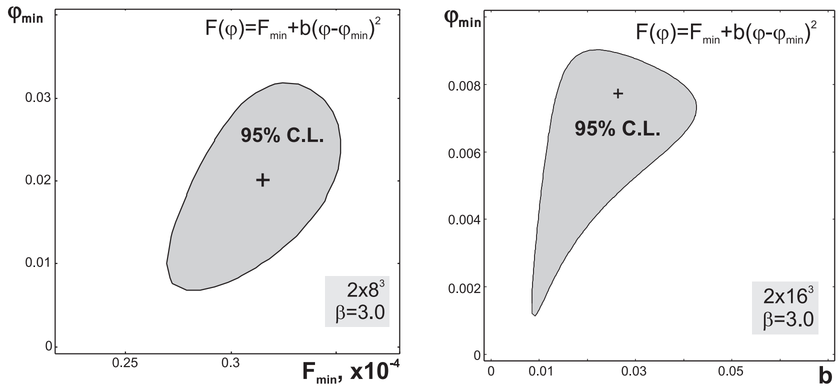

Figure 2. The 95% confidence level area for the parameters $F_{\rm min}$ and $\varphi_{\rm min}$, determining the free energy density dependence on the flux $\varphi_{\rm min}$ on lattice 2×8^3 for $\beta = 3.0$ (left figure). The 95% confidence level area for the parameters b and $\varphi_{\rm min}$, determining the free energy density dependence on the flux $\varphi_{\rm min}$ on lattice 2×16^3 for $\beta = 3.0$ (right figure).

3. The spontaneous vacuum magnetization and A_0-condensate

In this section we investigate an effective potential, taking into consideration the effects of a non-trivial A_0-condensate and chromomagnetic field [15]. This is realized by means of calculation of the partition function in a general covariant background field gauge with the background field providing both the chromomagnetic field and the non-trivial Polyakov loop. Then the data obtained in MC simulations are substituted in this effective potential. If the minimum value of it is negative and the potential is real, the common generation of condensates happens.

Within the used imaginary time formalism, the temporal direction in the Eucleadian space is periodic, with period $\beta = 1/T$, and the Polyakov loop $P_{\rm L}$, defined as the time-ordered exponential:

$$P_{\rm L}(\vec{x}) = T \exp\left[{\rm i}g \oint {\rm d}\tau\, A_0(\vec{x}, \tau)\right], \tag{10}$$

is a proper order parameter to describe the deconfinement phase transition. It is specified by a constant A_0 field, given in the fundamental representation by

$$A_0 = \frac{\phi}{g\beta}\frac{\tau_3}{2}, \tag{11}$$

where $0 \leqslant \phi \leqslant 2\pi$, τ_3 is Pauli matrix.

The trace of the Polyakov loop in the fundamental representation is

$$\mathrm{Tr}_F(P_{\rm L}) = 2\cos(\phi/2). \tag{12}$$

Unless the $Z(2)$ symmetry is spontaneously broken, the variable ϕ has to have the value π corresponding to $\mathrm{Tr}_F(P_{\rm L}) = 0$.

The one-loop contribution to the free energy has the form [15]

$$V^{(1)} = V_0^{(1)} + V_\pm^{(1)}$$
$$= \sum_n \frac{1}{\beta}\int \frac{{\rm d}^3\vec{k}}{(2\pi)^3} \log\left(\omega_n^2 + \vec{k}^2\right)$$

$$+\frac{1}{2}\sum_{m=0}^{\infty}\sum_{n,\pm}\frac{1}{\beta}\frac{gH}{2\pi}\int\frac{\mathrm{d}k_3}{2\pi}\log\left[\left(\omega_n-\frac{\phi}{\beta}\right)^2+2gH\left(m+\frac{1}{2}\pm 1\right)+k_3^2\right]$$
$$+\frac{1}{2}\sum_{m=0}^{\infty}\sum_{n,\pm}\frac{1}{\beta}\frac{gH}{2\pi}\int\frac{\mathrm{d}k_3}{2\pi}\log\left[\left(\omega_n+\frac{\phi}{\beta}\right)^2+2gH\left(m+\frac{1}{2}\pm 1\right)+k_3^2\right], \qquad (13)$$

where $\omega_n = 2\pi n/\beta$ is the Matsubara frequency, and the sum over n is over all integer values. The sum over m is the sum over the Landau levels.

By applying a standard technique of Schwinger to express the logarithms of (13) as an integral and performing the high-temperature expansion of the effective potential, we obtain

$$V^{(1)} = C_1\frac{(gH)^2}{8\pi^2} - \frac{(gH)^{3/2}}{4\pi^{3/2}\beta}\sum_{k=2}^{\infty}\frac{2^{2k}B_{2k}}{(2k)!}\Gamma(2k-3/2)$$
$$\times\sum_l\left(\frac{\beta^2 gH}{\beta^2 gH+(\phi-2\pi l)^2}\right)^{2k-3/2} + \frac{(gH)^2}{24\pi^2}(3-4\gamma) - \frac{2\pi^2}{45\beta^4}$$
$$+\frac{\phi^2}{3\beta^4}+\frac{\phi^4}{12\pi^2\beta^4}+\frac{gH}{2\pi\beta^2}\sqrt{-gH\beta^2+\phi^2}+\frac{g^2H^2}{12\pi\sqrt{gH\beta^2+\phi^2}}$$
$$-\frac{(gH\beta^2+\phi^2)^{3/2}}{3\pi\beta^2}-\frac{g^2H^2\log\left[\frac{\sqrt{-gH\beta}}{4\pi}\right]}{4\pi^2}+\frac{g^2H^2\log\left[\frac{\sqrt{gH\beta}}{4\pi}\right]}{12\pi^2}$$
$$-\frac{g^2H^2\phi^2\zeta[3]}{24\pi^4}-\frac{g^3H^3\beta^2\zeta\left[3,-\frac{\phi}{2\pi}\right]}{192\pi^4}-\frac{g^3H^3\beta^2\zeta\left[3,\frac{\phi}{2\pi}\right]}{192\pi^4}+O(\beta^2(gH)^3). \qquad (14)$$

Here $C_1 = -0.016\,46$, $\gamma \simeq 0.577\,216$ is Euler's constant, $\zeta(s)$ is the Riemann zeta-function and B_n are the Bernoulli numbers.

We performed the above-described lattice calculations and have obtained the values for the spontaneously generated field strength and the Polyakov loop of corresponding states. Then we considered expression (14) as a function of these parameters and studied its minimum. We have determined that for the temperatures $1/\beta = 100$–500 GeV, the field strengths $H = 10^{22}$–$10^{24}G$, and $\phi = 2.0$–3.11 the one-loop effective potential (14) is negative, $V^{(1)}(H,\phi,\beta) < 0$. Hence, it follows that either the gauge field or the A_0-condensate is spontaneously generated. Moreover, in the minimum of the effective potential for used values of the parameters the ratio $\mathrm{Im}V^{(1)}/\mathrm{Re}V^{(1)}$ is of order $\sim 10^{-10}$, that is, practically zero for numeric calculations. We conclude that the common generation of the constant Abelian chromomagnetic field and the non-trivial Polyakov loop results in the stable magnetized vacuum state. Of course, the one-loop potential does not exhibit the complete effective potential. However, it gives the main contribution which contains a larger imaginary part. If one adds the second next-to-leading contribution coming from ring diagrams, this part is cancelled [6]. So, the check of stabilization at the one-loop level is important.

4. Discussion

We have investigated the effect on common spontaneous generation of the chromomagnetic field and A_0-condensate in $SU(2)$-gluodynamics at high temperature. The first step was to show the possibility of the spontaneous generation of the chromomagnetic field at high temperature [12]. The obtained results are in a good agreement with those already derived in the continuum quantum field theory [6, 17] and in the lattice data analysis [11]. The actual values of the chromomagnetic field strength and the Polyakov loop were obtained from the same MC simulations. They allow us to conclude that these condensates, chromomagnetic field and A_0 have to be present in the deconfinement phase of QCD.

The developed approach joins the calculation of free energy functional $F(\phi)$ and the consequent statistical analysis of its minimum positions for various temperatures and flux values. In this way, the spontaneous creation of condensates is realized in lattice simulations.

The other important result of the present work is that the spontaneously magnetized state is stable. This indicates that a true vacuum at high temperature is formed from these condensates.

The investigated field configuration with constant fields is a solution to field equations without sources. Therefore, it can be spontaneously generated. At the same time, it is gauge non-invariant. Hence, one could believe that this configuration is a domain. A complete structure of whole space can be derived by using the requirement of gauge invariance for the space including domains with different orientations as is, in particular, assumed in the paper [19]. This point will be investigated separately.

In fact, the main goal of the present investigation was to confirm the results on the spontaneous vacuum magnetization discovered in continuum field theory [6]. As we have determined, this is really the case. The field strength generated at high temperature, which was estimated in continuum, coincides with the one obtained in the lattice simulations. This coincidence may serve as an argument in favor for this conclusion. In the present paper, the finite-size effects have not been investigated in detail. However, these effects are important near the phase transition temperature. They make it difficult to distinguish a first-order phase transition from a second-order one. In our case, the temperature is far from T_c. The fact that an external field penetrates the Coulomb phase is well known [13, 18]; so the only really new thing is that this field is spontaneously created. It was first observed in continuum [6], where the field strength of order g^4 in the coupling constant was established. Finite-size effects are not able to remove this result.

One could speculate that the lattice sizes 2×8^3 and 2×16^3 are not sufficient. However, these lattice sizes were used in [20]. The main aim of the present paper is to show a possibility of spontaneous generation of the chromomagnetic field at high temperature in lattice simulation, which was already investigated by perturbative methods [6, 10].

We would like to conclude that in the deconfinement phase, the condensates have to influence various processes that should be taken into consideration to have an adequate concept about this state.

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